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Solid-State YIG Serrodyne

DENIS C. WEBB, MEMBER, IEEE, AND ROBERT A. MOORE, MEMBER, IEEE

Abstract—**Theory and operation of a serrodyne based upon phase velocity modulation of magnetostatic modes in YIG are described. Sources of limitation on the spectral performance are evaluated in terms of measured device parameters. The most critical sources of unwanted spectral generation are flyback time, nonlinear current sawtooth, logarithmic phase variation, and variation in attenuation with applied magnetic bias. Design and operation of a *C*-band stripline device is described. All unwanted sidebands were suppressed by 22 dB over the desired signal output, limited primarily by the flyback time.**

INTRODUCTION

A MICROWAVE signal can be converted to a single sideband by sawtooth modulation of its phase velocity in a transmission or delay medium. Such a process is known as serrodyne frequency translation. It has proved useful in a wide variety of microwave applications [1] including Doppler simulation and correction, scanning of antenna arrays, and frequency shifting in microwave relay systems. The particular delay device that has received the greatest serrodyne application is the traveling wave tube. In the TWT serrodyne, the modulation is applied to the helix or cathode, causing a periodic bunching of the electron beam. Cumming [2] analyzed and tested this device in detail, developing and evaluating a set of performance criteria. With slight modification, these criteria can be applied to any phase modulable device such as the klystron, ferrite phase shifter [3] or crystal modulator [4], all of which have been used as serrodynes.

In this paper a *C*-band solid-state serrodyne is described. The frequency offset is obtained by using the magnetically variable phase velocity of magnetostatic modes propagating in a yttrium iron garnet rod. In the following sections, a brief outline of phase properties of magnetostatic modes in rods is first given, followed by design and performance characteristics of the YIG serrodyne. Spectral purity is discussed based on criteria developed by Cumming. Insertion loss and modulation power are also discussed. Minimal size, weight, and modulation power make the present device especially attractive for satellite application.

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D. C. Webb is with W. W. Hansen Laboratories of Physics, Stanford, Calif. 94305. He was formerly with the Microwave Physics Group, Aerospace Division, Westinghouse Defense and Space Center, Baltimore, Md. 21203
R. A. Moore is with the Microwave Physics Group, Aerospace Division, Westinghouse Defense and Space Center, Baltimore, Md. 21203

PHASE PROPERTIES OF YIG RODS

It was noted above that the traveling wave tube relies upon modulation of the electron velocity to achieve frequency translation. As Cumming pointed out, when beam modulation is employed, the wave velocity is established at the time the waves enter the device. By contrast, in the magnetostatic mode device modulation is applied continuously during transit of the rod. However, Cumming's analysis is still valid as long as the ratio of transit time to the modulation period is small. This criterion is met in the experiments to be described, being about 4 percent in the operating region of interest. An important practical distinction, the YIG serrodyne in the magnetostatic mode is a narrow band —though electronically tunable—device. This is in contrast to the TWT device for which the device is only mildly frequency dependent.

Another difference between the TWT and YIG serrodynes is a dissimilarity in phase and group velocity characteristics. In the TWT both characteristics are similar since the device is usually operated in a linear portion of the $\omega-\beta$ diagram. Both phase and group velocity are nearly proportional to the square root of the applied voltage. Fundamentally, serrodyne action involves the phase velocity or, more generally, the phase delay which Cumming calls the transit time. Cumming uses the symbol τ for phase delay whereas here this symbol is used for group delay in keeping with delay-line practice.

In the YIG device, however, phase and group velocity characteristics follow distinctly different relations. Group velocity or delay characteristics have been investigated in detail within the past few years [5]–[8]. Recently Damon and van de Vaart [7] showed that the group delay through the rod has the form

$$\tau = \frac{X_i(1+\rho_a^2)^{5/4}}{\gamma\sqrt{3}\,\rho_a^2\Delta H} \tag{1}$$

where

X_i is a root of $J_0(X_i)=0$

$\rho_a = D/l =$ aspect of diameter to length ratio of the rod

$\gamma =$ gyromagnetic ratio

$\Delta H = (\omega/\gamma) - H_i$ where H_i is the axial magnetic field at the center of the rod. $H_i = \omega/\gamma$ defines the field for which the group delay diverges.

$\tau =$ one-way group delay through the rod. One-way group delay is given here since this is the mode in which the serrodyne operates. The magnetic field bias is parallel to the rod.

Using the same basic assumptions as Damon and van de Vaart [7], the total phase shift through the rod is given by the expression

$$\phi = \frac{X_i(1+\rho_a^2)^{5/4}}{\sqrt{3}\,\rho_a^2} \log \frac{6\rho_a^2}{(\rho_a^2+1)^{5/2}} \frac{4\pi M_s}{\Delta H} \tag{2}$$

where the magnetic field bias is parallel to the rod axis. Note that in contrast to the delay time which is inversely proportional to ΔH, the phase shift varies logarithmically with this field difference. It is evident that the total phase shift also diverges for $\Delta H=0$.

Experiments were conducted to verify the relationships given in (1) and (2). The results are summarized in Fig. 1. Group delay results are in accord with Damon; that is, experimental points form a slightly less dispersive curve than is predicted analytically. The phase-shift characteristics are measured in terms of the difference field δh defined such that

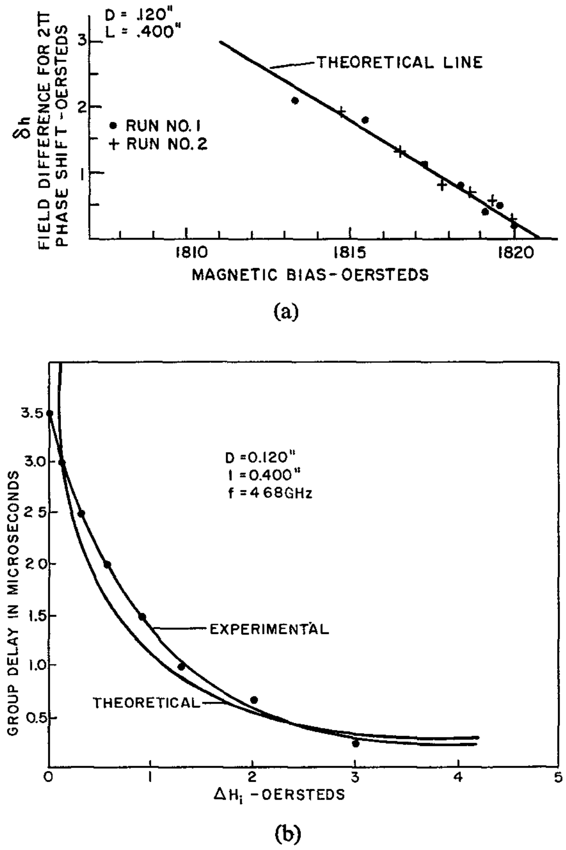


Fig. 1. Theoretical and experimental curves of (a) phase shift, (b) group delay.

$$\phi(\Delta H - \delta h) - \phi(\Delta H) = 2\Pi.$$

Phase measurements agree very well with predicted value, the only adjustable parameter being the value of the field for which the group and phase delay diverge. The value of this field was only 7 Oe from the theoretically predicted value. Thus, the analytical model assumed appears to predict experimental characteristics sufficiently well to serve as the basis for device design.

Periodic Modulation

Application of a periodic magnetic field variation in addition to the dc bias field will result in the generation of frequency sidebands. By applying a sawtooth modulation of the proper amplitude, the incident signal can be converted to a single sideband, offset from the original frequency by the modulation frequency. This is illustrated below, assuming an exponential phase variation in time. Assuming the total phase ϕ varies in the sawtooth fashion as shown in Fig. 2, the wave function can be expressed as

$$A = A_0 \exp\left\{i\left[\left(\omega + \frac{\Delta\phi}{\tau_m}\right)\tau - \phi_0\right]\right\} \tag{3}$$

where A_0 is the signal amplitude. If the phase is displaced by 2π, i.e., $\Delta\phi = 2\pi$, each period and the flyback time is negli-

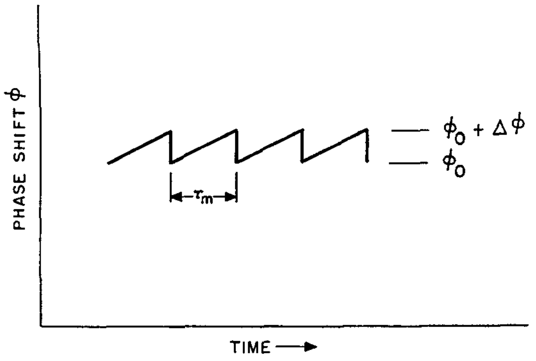


Fig. 2. Single sideband generation through sawtooth modulation.

gible, the transmitted phase characteristic is as though the phase were increasing linearly at the rate of 2π radians for each period τ_m. The frequency is shifted or serrodyned by τ_m or f_m. Note that it is the phase rather than the group characteristic that is significant. The significance of a variation from the linear phase shift and periodic-amplitude characteristic and their effect and limitation on operating characteristics for serrodyne applications are further treated in the next section.

Operating Considerations

Sideband Suppression

In practice, total conversion to a single sideband frequency cannot be achieved and spectral elements spaced at integral multiples of the modulation frequency will be present. They arise because of departures from ideal sawtooth phase modulation, amplitude variations, and other factors enumerated and evaluated by Cumming [2].

The predominant factors limiting the spectral purity of the YIG serrodyne were:

1) nonzero flyback time,
2) nonoptimum sawtooth amplitude,
3) amplitude modulation, and
4) nonlinear sawtooth phase variation.

Quantitatively, a measure of serrodyne performance in sideband suppression can be defined as [2]

$$S = 20 \log \frac{\text{(output signal amplitude at the desired frequency)}}{\text{(output signal amplitude at an undesired frequency)}} \quad (4)$$

where the output at an undesired frequency is that due to all causes. The expressions for the suppression given in this section are taken for the most part from Cumming, modifications being made when necessary to fit specific properties of the present device. Although in practice a number of limiting factors will be competing, each will be treated separately and the combined effect discussed subsequently. For example, if two had about the same magnitude, the suppression due to both would be approximately 3 dB greater than each individually.

In practice, it is of course impossible to generate a sawtooth waveform that returns instantaneously from the peak value to zero. In Cumming's TWT modulator [2], the limitation is the rate at which the voltage across the stray capacitance can be discharged through a tetrode. In the YIG device, the flyback time is limited by the switching time of the transistor multivibrator circuitry. Quantitatively, the effect of the finite fractional flyback time F on the suppression is given by [2]

$$S = -20 \log F. \quad (5)$$

It was noted previously that the sawtooth amplitude must be such to cause a 2π phase variation per period. Nonoptimum adjustment results in sideband generation as given by [2]

$$S = -20 \log \frac{\pi(X-1)}{\sin \pi X} \quad (6)$$

where X= fractional deviation from optimum.

The losses throughout YIG are strongly field dependent; thus, any modulation of the field will cause a corresponding amplitude modulation. Following the technique developed by Cumming [2], the suppression can be closely approximated by

$$S = -20 \log_{10} 0.0367\, \alpha_m \tau_0 \frac{\delta h}{\Delta H} \quad (7)$$

as shown in Appendix I where

α_m is the loss per microsecond delay,
τ_0 is the group delay at the bias field ΔH, and
δh is the field difference that must be applied for a 2π phase shift.

Results are similar to the parabolic case of Cumming [1], being about 0.5 dB worse for the current set of device parameters in the range of interest.

Nonlinearities in the phase variation similarly can cause unwanted sidebands. The magnitude is given by [2]

$$S = 20 \log \frac{A}{\pi d} \quad (8)$$

where

d is the maximum deviation from a best fit straight line approximation, and
A is the maximum amplitude of the sawtooth.

This places a requirement on the linearity of the applied current waveform.

It was also previously noted that the phase changes logarithmically with field change. Thus, the phase variation will be nonlinear even with a linearly applied field variation. The suppression for this effect is closely approximated by

$$S = -20 \log \frac{\pi}{16} \frac{\delta h}{\Delta H} \quad (9)$$

as shown in Appendix II.

Modulation Requirements

Several methods of modulation might be used, the modulation power being dictated by the particular one selected. We used a driven sweep, supplying the total energy for the phase shift each cycle. This results in an upper limit of the power required. Quantitatively

$$P = \tfrac{1}{2}L(\Delta I)^2 f \qquad (10)$$

where L is the coil inductance and ΔI is the sawtooth current amplitude. Combining the field required shown in Fig. 1 with the expressions for inductance of and field in a "long" solenoid, the power may be written in the form

$$P = 5.6\Delta H^2 V f (10)^{-9} \qquad (11)$$

where

P = power, W,
V = coil volume, in^3, and
f = frequency, Hz.

For operating parameters of the 150 kHz model to be described, P is less than 0.1 μW. Thus, in practice the standby and switching power absorbed in the modulation circuitry rather than that used in effecting the phase shift will determine the total modulation power required.

The current needed to provide a 2π phase shift depends upon the coil and rod geometry and the bias field. It can be conveniently expressed as

$$\Delta I\sqrt{L} = 3.6\delta h \sqrt{\frac{r^2 + \left(\frac{l}{2}\right)^2}{6r + 9l + 10b}} \qquad (12)$$

where

r = coil radius,
l = coil length,
b = coil thickness (multiply wound coils), and
L = inductance in henries.

For the parameters in our experimental model

$$\Delta I = 4.8\Delta H$$

ΔI being in milliamperes and ΔH in oersteds.

Insertion Loss

The two contributing elements to the insertion loss are transmission attenuation and transduction efficiency. Transduction efficiency depends upon the exciting geometry while the transmission attenuation is a characteristic of the material. At a fixed frequency, the latter is proportional to the group delay through the rod. Experimental group delay and insertion loss data taken on the rod used in the current device are presented in Figs. 1 and 3, respectively.

The attenuation is best approximated by the line $\alpha_m = 12.2$ dB/μs. This agrees well with the $2f+3$ dB/μs measurement of Strauss [10] and subsequent measurements by Sparks [11]. Although no systematic study of means to reduce transduction loss was made, total insertion losses as low as 13 dB were achieved. It was found to be very sensitive to placement of the rod in the resonator.

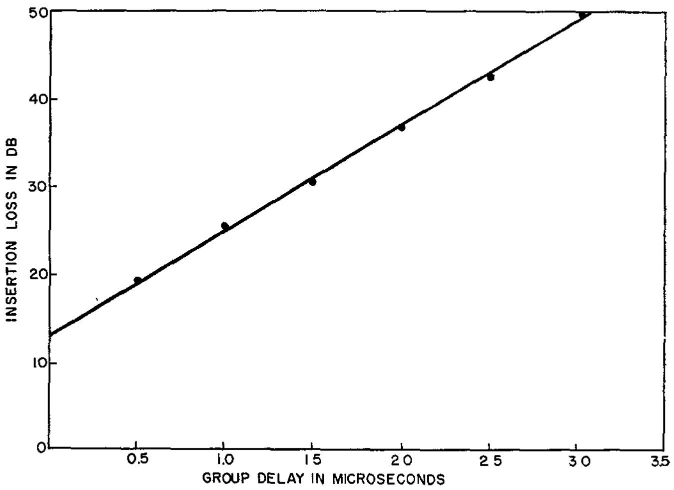


Fig. 3. Insertion loss of magnetostatic mode in a YIG rod.

Experimental Model

Microwave Circuitry

Microwave circuitry for the serrodyne consisted of two orthogonal resonators coupled by the YIG rod. The model used in the current set of experiments is shown schematically in Fig. 4 and photographically in Fig. 5. Capacitive coupling to the resonators was achieved by extending the center conductor of the coaxial line. The tuning screws provided any necessary fine tuning adjustment. The resonators and YIG rod were supported by a copper plated polystyrene block to minimize leakage and eddy current losses. Measured leakage was at least 70 dB below the incident power level. The unloaded cavity Qs were about 450. An electromagnet was used to provide the dc bias.

Modulation Circuitry

Sawtooth current modulation was applied to a 5 μH coil wrapped around the rod as shown in Fig. 4. It was derived from the emitter of one of the transistors in an emitter-coupled astable multivibrator. Two emitter follower stages were used in addition to achieve the required current amplitude. The actual current waveform applied to the coil is shown in Fig. 6. The peak to peak variation is approximately 14 mA.

Spectral Characteristics

Figure 7 shows the spectral characteristics resulting from applying the sawtooth current waveform to the YIG rod, displayed on the scope face of a spectrum analyzer. The carrier frequency is 4.68 GHz and the offset frequency 150 kHz. The width of the line is determined by the resolution bandwidth of the analyzer. All unwanted spectral elements could be simultaneously suppressed 22 dB below the desired offset frequency. Specific sideband frequencies, including the input

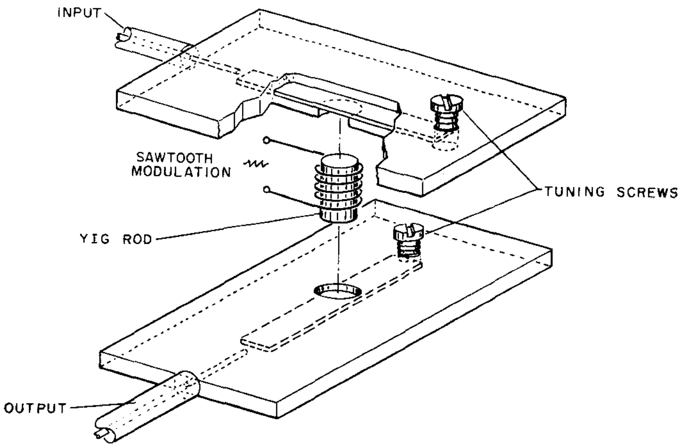


Fig. 4. Exploded view of stripline serrodyne.

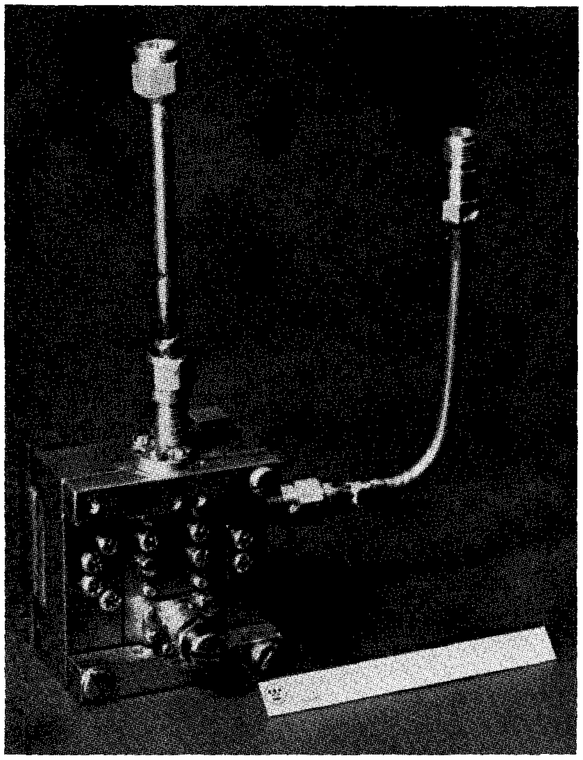


Fig. 5. Actual stripline model.

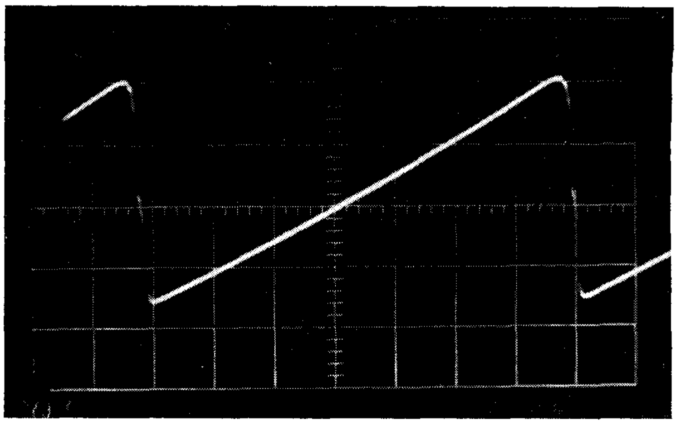
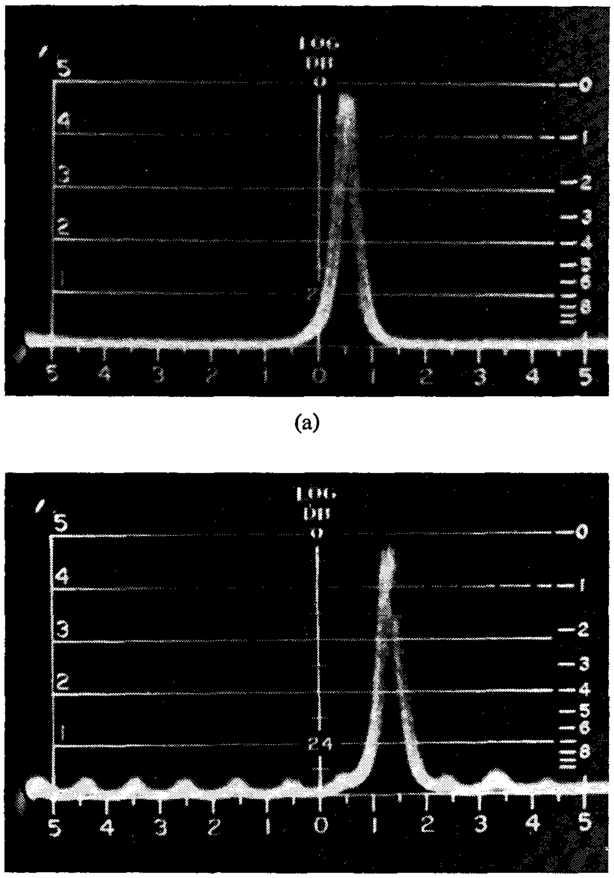


Fig. 6. Sawtooth waveform derived from an emitter coupled multivibrator.



(a)

(b)

Fig. 7. Experimentally obtained spectral characteristics. (a) Without sawtooth modulation. (b) With sawtooth modulation.

frequency, could be suppressed to 30 dB below the desired sideband frequency by adjusting the dc and modulation fields.

Based on expressions presented earlier in this paper and measured properties of the device, individual contribution to unwanted spectral lines are:

1) flyback time—23.2 dB,
2) periodic amplitude variation—29 dB,
3) nonlinear current waveform—30 dB, and
4) logarithmic phase variation—24.5 dB.

Combining these contributions leads to an unwanted spectrum level of -20 dB. Experimentally the value achieved was slightly better than this, and it is probable that the difference was caused by a cancellation of the contributions. The ability to null specific lines is further evidence that this is occurring.

Thus, although phase and amplitude nonlinearities would at first appear to present serious difficulties, both experimental and analytical results point to the flyback time as the chief cause of unwanted spectral generation. Reduction of the phase and amplitude nonlinearity could be further achieved by using samples with a lower aspect ratio.

Conclusions

In conclusion, it has been shown that magnetostatic mode propagation in YIG rods can be effectively utilized in constructing a serrodyne. A C-band stripline version has been tested and suppression of all undesired sideband frequencies to 22 dB achieved, limited primarily by the flyback time. This is in good agreement with theoretical predictions, based on experimentally measured device parameters. Modulation power consumed in the serrodyne process is negligible and total insertion loss is less than 17 dB.

By using stripline circuitry and permanent magnets, light compact devices can be achieved, making the YIG serrodyne attractive for satellite applications.

Appendix I

Presented in this section is a derivation of spurious sideband generation due to attenuation dependence of the magnetic field. Assuming that this factor alone limits the spectral purity, the time dependent delay and field are related by

$$[\tau_0 + \Delta\tau(t)]\left[\Delta H - \delta h \frac{t}{\tau_m}\right] = \tau_0 \Delta H \tag{13}$$

where τ_0 is the time independent delay and $\Delta\tau(t)$ is the delay as a function of time from the beginning of each period.

The time varying delay can be written as

$$\Delta\tau(t) = \tau_0 \frac{\frac{\delta h}{\Delta H}\frac{t}{\tau_m}}{1 - \frac{\delta h}{\Delta H}\frac{t}{\tau_m}} \cdot \tag{14}$$

Since the attenuation in decibels per microsecond α_m is constant, the time dependent attenuation can be written

$$\alpha(t) = \alpha_m \tau_0 \frac{\delta h}{\Delta H}\frac{t}{\tau_m} \Big/ \left[1 - \frac{\delta h}{\Delta H}\frac{t}{\tau_m}\right]. \tag{15}$$

In terms of amplitude

$$\frac{A}{A_0} = 10 \exp\left[-\frac{\alpha_m \tau_0}{20} \frac{\frac{\delta h}{\Delta H}\frac{t}{\tau_m}}{1 - \frac{\delta h}{\Delta H}\frac{t}{\tau_m}}\right].$$

By letting

$$\frac{\alpha_m \tau_0 \log_e 10}{20} = C$$

and

$$\frac{\delta h}{\Delta H}\frac{t}{\tau_m} = X,$$

the amplitude can be expanded in the form

$$\frac{A}{A_0} = 1 - CX + (\tfrac{1}{2}C^2 - C)X^2 - (\tfrac{1}{6}C^3 - \tfrac{1}{2}C^2 + C)X^3 + \cdots . \tag{16}$$

Cumming [2] shows that the suppression can be written as

$$-20 \log |M_n(q, r)| \tag{17}$$

where

$$M_n(q, r) = M_n(0, r) + iM_n' q_1(0, r) - q_2 M_n''(0, r) + \cdots \tag{18}$$

and

$$M_n(0, r) = \frac{\sin X}{X} \cdot$$

q_i represents the coefficient of the ith exponent term, e.g.,

$$q_1 = -C\frac{\delta h}{\Delta H}; \quad q_2 = \left(\frac{\delta h}{\Delta H}\right)^2 (\tfrac{1}{2}C^2 - C),$$

etc. Combination and simplification of (16) through (18) yields

$$S = -20 \log_{10} 0.0367 \alpha_m \tau_0 \frac{\delta h}{\Delta H} \cdot$$

Higher-order terms contribute only a few tenths of a decibel, thus for most applications are negligible.

Appendix II

In this section the effect of the logarithmic-phase characteristics upon sideband suppression is detailed.

$$\phi(t) = C_1 \log \frac{4H_2}{\Delta H - \delta h(t)} \tag{19}$$

or

$$\phi(t) = \phi_0 - C_1 \log\left(1 - \frac{\delta h}{\Delta H}\frac{t}{\tau}\right). \tag{20}$$

Expanding in a Taylor series

$$\frac{\Delta\phi_2(t)}{C_1} = -\frac{\delta h}{\Delta H}\frac{t}{\tau} + \frac{1}{2}\left(\frac{\delta h}{\Delta H}\frac{t}{\tau}\right)^2. \tag{21}$$

The remainder of the series can be dropped with negligible effect upon the spectral properties; this method of evaluation is also based on Cumming's work [2].

A straight line passing through $t=0$ and $t=\tau$ has the form

$$\frac{\Delta\phi_s}{C_1} = \left[-\frac{\delta h}{\Delta H} + \frac{1}{2}\left(\frac{\delta h}{\Delta H}\right)^2\right]\frac{t}{\tau} \cdot \tag{22}$$

The difference of the logarithmic ordinate and the straight-line ordinate can be derived from (21) and (22) to be

$$\frac{\Delta\phi_2 - \Delta\phi_s}{C_1} = \frac{1}{2}\left(\frac{\delta h}{\Delta H}\right)^2\left[\left(\frac{t}{\tau}\right)^2 - \frac{t}{\tau}\right]. \quad (23)$$

The maximum difference occurs at $t=\tau/2$ and is $\frac{1}{8}(\delta h/\Delta H)^2$. Thus a straight line passing equidistant from the center and end points will differ $\frac{1}{16}(\delta h)^2/(\Delta H)$ from the logarithmic curve. The suppression is therefore given by

$$S = 20\log\frac{A}{\pi d} = -20\log\frac{\dfrac{\delta h}{\Delta H}}{\dfrac{\pi}{16}\left(\dfrac{\delta h}{\Delta H}\right)^2}$$

$$= -20\log\frac{\pi}{16}\frac{\delta h}{\Delta H}. \quad (24)$$

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Correspondence

A Stability Criterion for Tunnel Diode Amplifier

Abstract—**A relatively simple stability criterion is proposed for bandpass tunnel diode amplifiers. The results predicted from this criterion agree very closely with the results obtained from the analog computer simulation of an experimental amplifier. The derived criterion is solved to determine the limitation on the diode series inductance as a function of the diode negative resistance, and the results are plotted for a wide range of diode parameters and amplifier gains.**

The successful design of a tunnel diode amplifier depends to a large extent on the proper analysis of amplifier stability. Unfortunately analytical solutions of the stability of practical amplifier circuits are usually complicated. Consequently, stability criteria which are derived for simple diode circuits are sometimes used for preliminary design of amplifiers. One such widely used criterion is that derived for a circuit consisting of a tunnel diode terminated in a pure resistance, R_0. In terms of the diode parameters this criterion is given by

$$\frac{L_S}{RC} < (R_0 + R_S) < R \quad (1)$$

where R is the magnitude of the diode negative resistance, C is the junction capacitance, and R_S and L_S are the parasitic series resistance and inductance, respectively. This criterion as well as others, referred to as optimum, such as those given by Frisch,[1] Markowski and Davidson,[2] Davidson,[3] and Smilen and Youla,[4] would, in many cases, give too optimistic results when used in the design of bandpass tunnel diode amplifiers. For example, in the case of an S-band shunt-tuned amplifier, the limiting value of L_S for stable amplification was found to be as much as one order of magnitude smaller than the value predicted by (1).

In this correspondence a stability criterion which is relatively simple and is more suitable for amplifier design is proposed. The proposed criterion differs from the other criteria in that the circuits analyzed include the shunt tuning inductance as shown in Fig. 1. The shunt inductance and the diode junction capacitance are assumed to be resonant at the amplifier midband frequency, f_0. The stability of the proposed circuit was compared with that of an experimental amplifier (see Fig. 2), by simulating the two circuits on the analog computer. The results agreed very closely as shown in Fig. 3. The proposed criterion is derived below.

The characteristic equation of the circuit of Fig. 1 can be readily shown to be given by

$$a_3p^3 + a_2p^2 + a_1p + a_0 = 0 \quad (2)$$

where

$$a_0 = R_0(R - R_S)$$
$$a_1 = L_S(-R_0) + L(R - R_0 - R_S) + R\ R_0R_SC$$
$$a_2 = L_S(R_0RC - L) + L(R\ R_0C + R\ R_SC)$$
$$a_3 = L\ L_SR\ C.$$

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